Building the Mathematical Model of the Hybrid Manipulator

INTRODUCTION

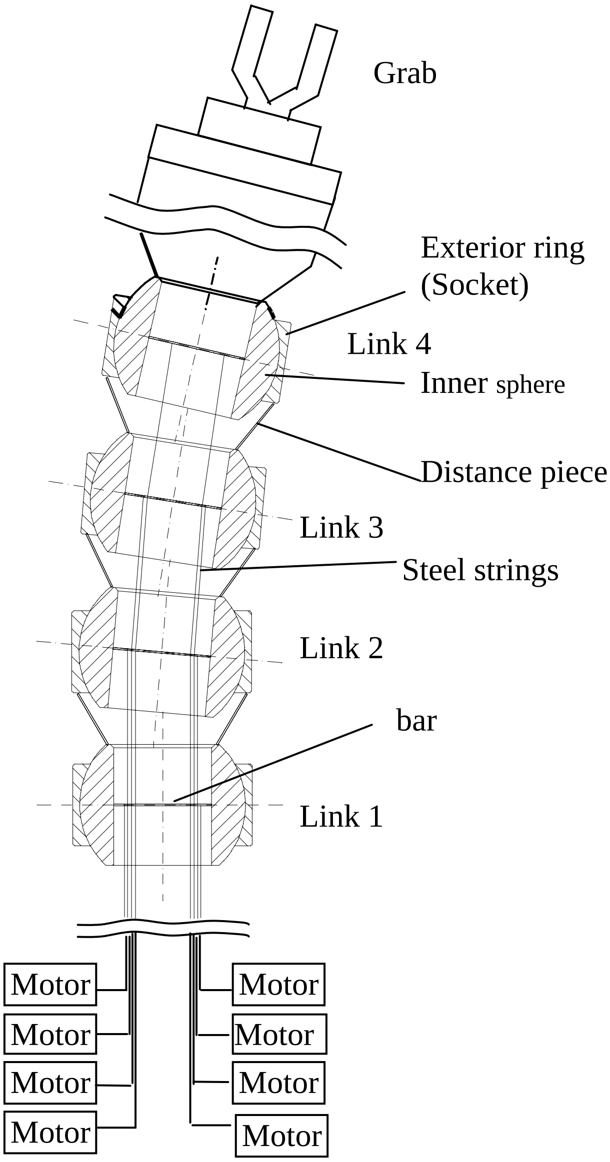

Fig.1: The new type of manipulator

Hybrid multi-joints manipulators typically have a chain structure and can move in two- and three-dimensional ways. The new type of manipulator shown in Fig. 1 combines a large number of joints to improve flexibility. Thus there will be many areas of application in industry, medicine, micro assembly and others.

Our multy-joints hybrid manipulator is made from plastic sphere links. Each link consists of four parts: the inner sphere, the horizontal metal bar at the center of the sphere, the distance piece and the exterior ring (the socket for a next link). On a base of our manipulator are situated motors which drive the links. Each link is connected with a motor by a pair of strings whose middle is fixed to the rotor's surface and two ends are fixed to the bar. Strings that drives next link are led through special holes in the previous links bar.

The advantages of our manipulator are: big working envelope, flexibility and high rigidity. Also the manipulator can be easily controlled by the strings according to the task. The first disadvantage of our manipulator is the elasticity of strings which reduces the accuracy of the manipulator control. The second disadvantage of our manipulator is the unknown value and the unknown dispensing of the links friction.

To solve these problems and to analyse and improve the dynamical behaviour of the manipulator mathematical models of the complete system have to be built up.

The mathematical model of the manipulator was made in the Dynamic Modeling Laboratory (Dymola) using the Modelica Multi-body library. The main problems of manipulator modelling are:

1. Measurements and approximation of links and rotors friction.
2. Building model's objects and links between them.

DESCRIPTION OF THE ONE-JOINT MANIPULATOR MODEL

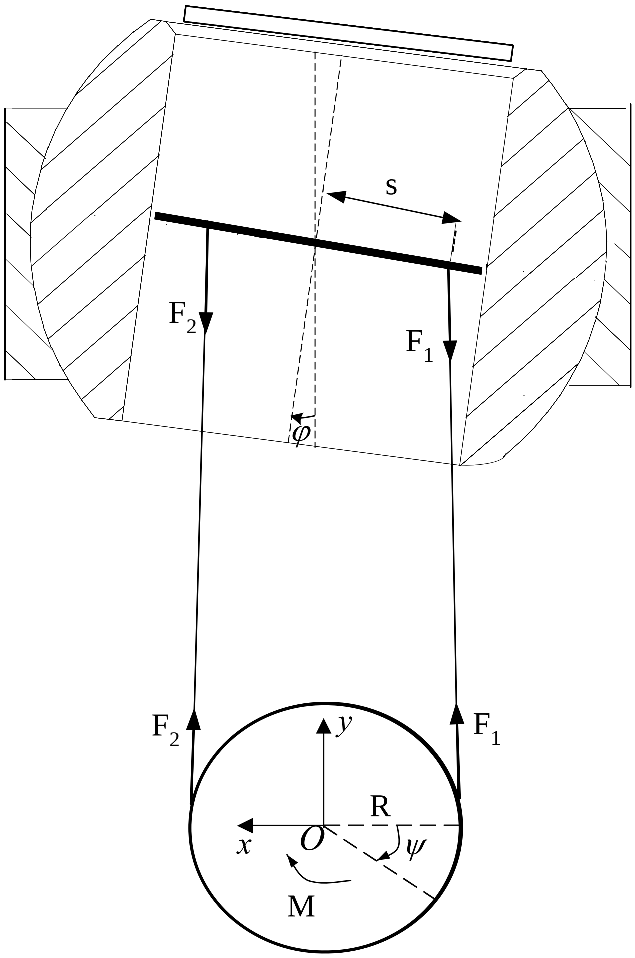

Fig. 2: The one-joint manipulator

Let's describe the motion of our multi-joints manipulator in the case of the one-joint manipulator shown in Fig. 2. Trying to reduce the comlexity of the model we assume that the link consitsts only of the inner sphere and the exterior ring. So, the link's mass center is situated on the center of the sphere.

Let Oxy denote the inertial frame with centre in the centre of the rotor. The torque M rotates the rotor with radius R on the angle ψ. So, when the rotor rotates the string stretches and the force on the string F appears. That force rotates the link on the angle φ. Let s denote the distance from the string to the centre of the link. This equation describes the elongation of the string Δl:

$$\Delta l = R \cdot \psi - s \cdot \sin \varphi .$$

Let F_1 be the force on the right end of the string, F_2 the force on the left end, and f the pre-stress acting on the string. So, we get this system of equations which describes the forces on the ends of the string:

$$F_1 = \begin{cases} f + \Delta l \cdot k & \text{when} \quad \Delta l > 0 \\ f & \text{when} \quad \Delta l < 0 \end{cases}$$

$$F_2 = \begin{cases} f & \text{when} \quad \Delta l > 0 \\ f - \Delta l \cdot k & \text{when} \quad \Delta l < 0, \end{cases}$$

where k is the string constant.

Let m_{fr} be the link friction torque, M_{fr} the rotor friction torque, J_r the moment inertia of the rotor, and J_l the moment inertia of the link. So, the rotor's and link's movement can be described with the following two equations:

$$F_1 \cdot R - F_2 \cdot R = M - M_{fr} - J_r \ddot{\psi}$$

$$F_1 \cdot s - F_2 \cdot s = J_l \ddot{\varphi} - m_{fr} .$$

We experimentally obtain the string constant $k=96000 N/m$.

APPROXIMATION OF LINK AND ROTOR FRICTION

The diagram shown in Fig. 3 illustrating measured values of the link friction torque m_{fr} for different values of the force in the link F_L and the link angular velocity ω:

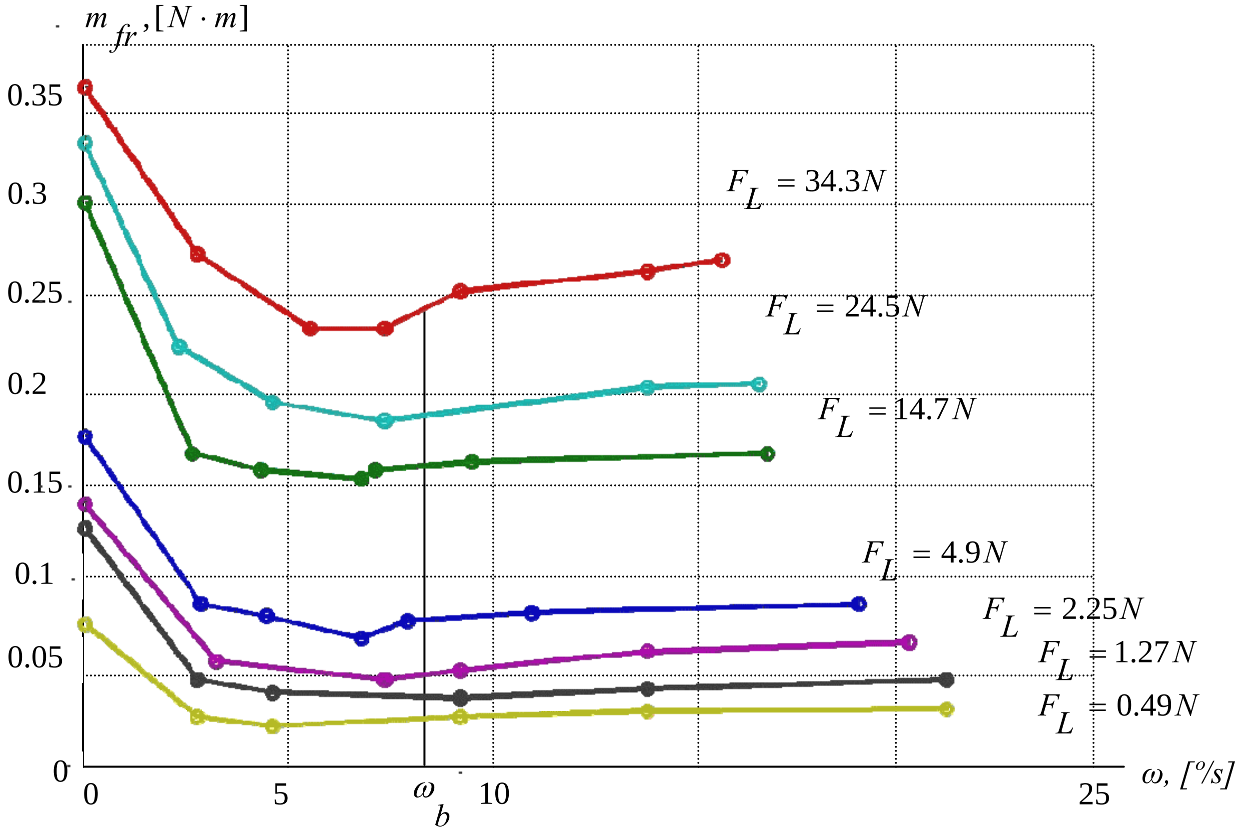

Fig. 3: Measured values of the link friction torque m_{fr}

The previous diagram shows that when the link angular velocity ω changes from 0 to the border of the Striebeck effect ω_b (=8 °/s) then the link friction torque m_{fr} changes in a parabolic way. With the help of the four experimental constants $R1$, $R2$, $\sigma 1$, $\sigma 2$ we can approximate experimental curves on the diagram:

$$m_{fr} = \begin{cases} \dfrac{z_1 - z_2}{\omega_b}\omega^2 + \dfrac{2(z_2 - z_1)}{\omega_b}\omega + z_1 & \text{when} \quad \omega \le \omega_b \\ z_2 + k_l(\omega - \omega_b) & \text{when} \quad \omega > \omega_b \end{cases}$$

$$z_1 = \begin{cases} \dfrac{\sigma_1}{\sqrt{R_1}}\sqrt{F_L} & \text{when} \quad F_L \le R_1 \\ k_d(F_L - R_1) & \text{when} \quad F_L > R_1 \end{cases}$$

$$z_2 = \begin{cases} \dfrac{\sigma_2}{\sqrt{R_2}}\sqrt{F_L} & \text{when} \quad F_L \le R_2 \\ k_d(F_L - R_2) & \text{when} \quad F_L > R_2. \end{cases}$$

Experimentally we get this constants values:

$$\begin{cases} \sigma_1 = 0.295 \\ \sigma_2 = 0.14 \\ R_1 = 10 \\ R_2 = 14. \end{cases}$$

The diagram shown in Fig. 4 illustrating experimental data m_{fr} and our approximation for $F_L = 24.5N$:

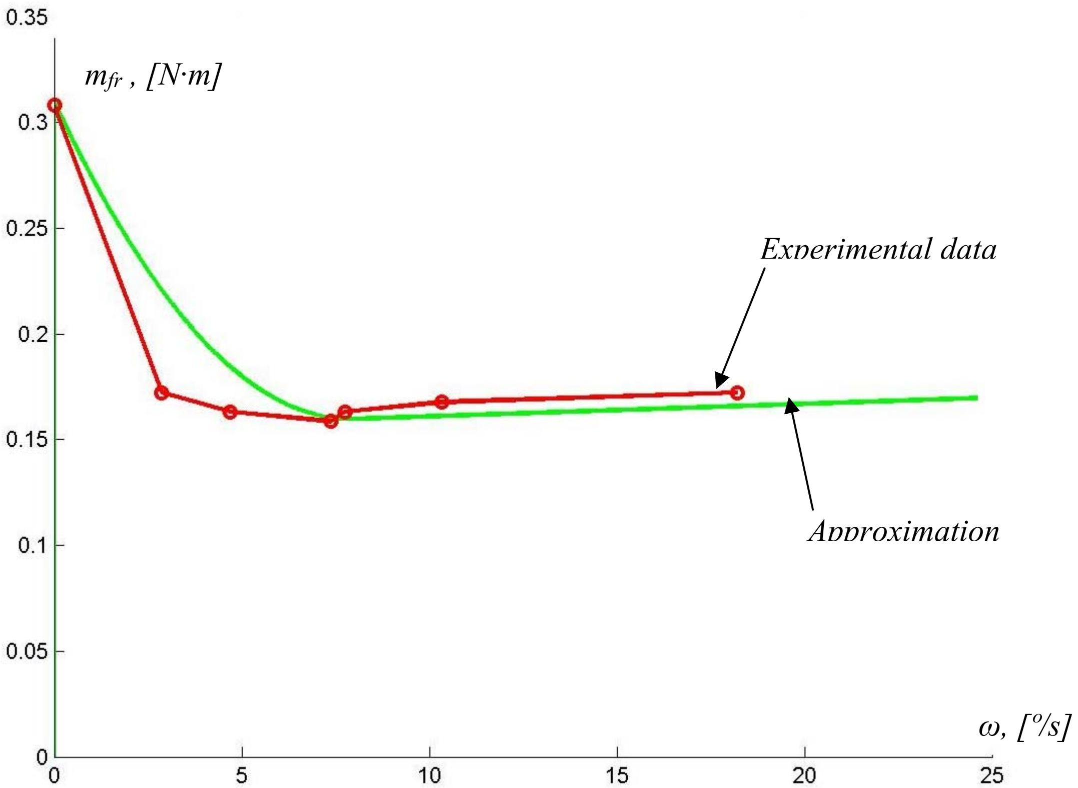

Fig.4: The experimental data and the approximation of the link friction torque when $F_L = 24.5N$

Rotor friction torque M_{fr} is approximated in our model according to:

$$M_{fr} = M_{fr}^A \, sign(\dot{\psi}),$$

where M^A_{fr} is a constant.

EXPERIMENTAL DATA

In our experiment the torque M is a periodical time function:

$$M = \sin(10 \cdot 2\pi \cdot t),$$

where t denotes time.

The time dependency of the rotation angle of the link φ and the rotation angle of the rotor ψ is shown in Fig. 5.

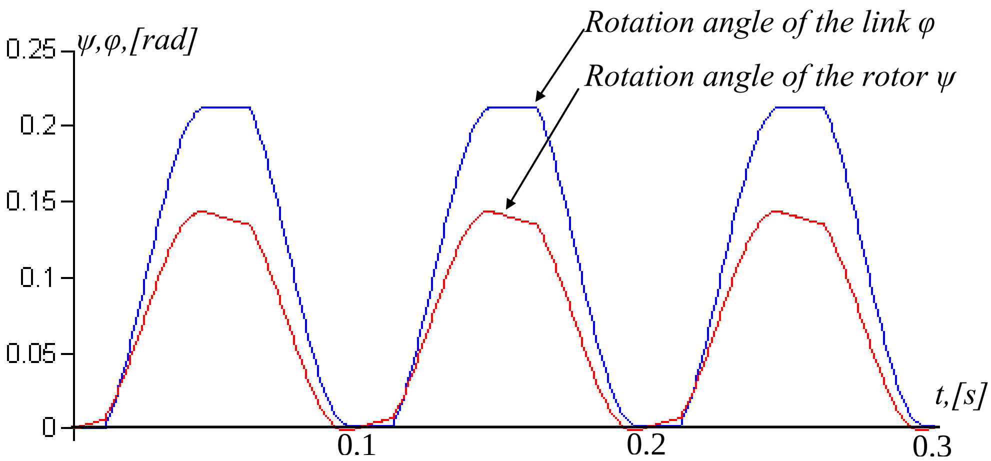

Fig. 5: Time dependency of the rotation angle of the link φ and the rotation angle of the link φ

This diagram shows that the elasticity of strings does not change significantly the behaviour of our model. It happens because of the big value of string constant k for a short string.

The peaks of the curves on the diagram are truncated – these are areas where the force on the string is less than the force of the links friction.

Time dependency of the angular velocity of the link ω and the angular velocity of the rotor v is shown in Fig. 6.

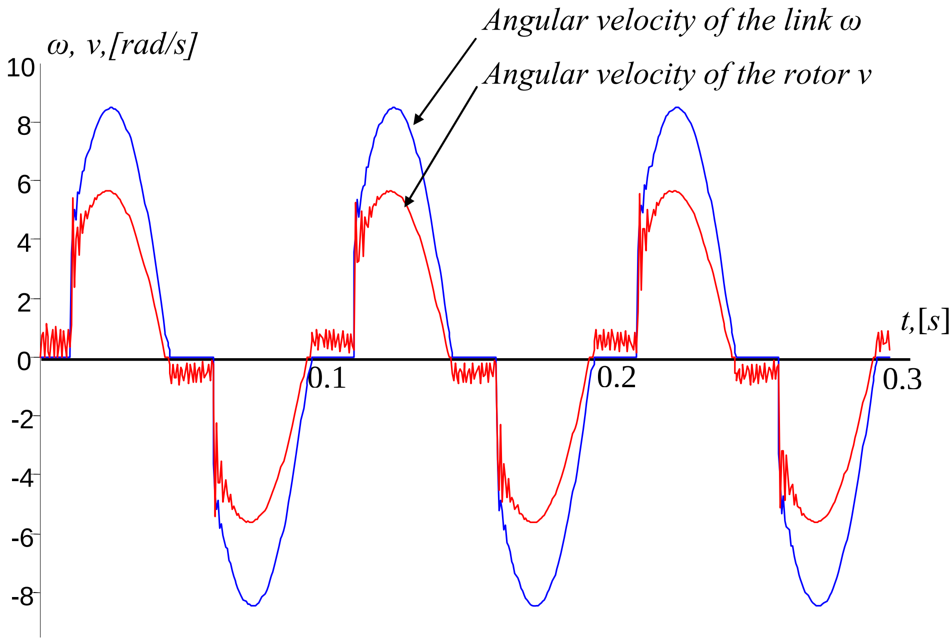

Fig.6: Time dependency of the angular velocity of the link ω and the angular velocity of the rotor v

This diagram shows that link does not move and rotor oscillates, when the modulus of the torque is small. When the modulus of the torque increases, the link and the rotor jump. It happens because the static friction is significantly larger than dynamic friction.

After the jump the angular velocity of the rotor and the angular velocity the link fluently increase because of the torque is increasing.

CONCLUSIONS

The building of the mathematical model of the hybrid manipulator with plastic links is a complex problem. For the correct mathematical description of our model we need to solve some subproblems: the measurement and the approximation of the links friction torque, the measurement of the string constant, the approximation of the rotor friction torque.

The simulation experiments with the mathematical model shows that the links friction significantly changes the behaviour of the manipulator movement. In the next steps further investigations are necessary to analyse the effect of friction in the case of multi-joint manipulator and to develop a control strategy for the link movement.